**Using Recursion in Models and Decision Making: Recursion Using**

**Rate of Change**

IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

**1.** Consider the exponential function . Fill in the table of values for the function, and

find the rate of change between consecutive values in the function (Δ***y***). What pattern do

you see for Δ***y*** change?

|  |  |  |
| --- | --- | --- |
| ***x***  | ***y*** | Δ***y*** |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |

**2.** In Student Activity Sheet 3, you learned about the sometimes fatal antibiotic-resistant

staph bacteria methicillin-resistant *Staphylococcus aureus* (MRSA) growing in an agar

dish. The initial area occupied by the bacteria in the agar dish is 2 square millimeters,

and they increase in area by 20% each week. The table below gives the area of the

bacteria over several weeks. Use the table to describe the rate of growth of the area

of the bacteria (Δ***a***).

|  |  |  |
| --- | --- | --- |
| ***x*** | ***a*** | ***Δa*** |
| 0 | 2 |  |
| 1 | 2.4 |  |
| 2 | 2.88 |  |
| 3 | 3.456 |  |
| 4 | 4.147 |  |
| 5 | 4.977 |  |

**3.** Suppose a quantity increases at a rate proportional to the quantity, and the constant of

proportionality is 0.2. The initial quantity is 2. Write a difference equation that describes

the statement above and find several values of this quantity.

|  |  |  |
| --- | --- | --- |
| ***x***  | ***y*** | Δ***y*** |
| 0 |  | 2 |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |

**4.** Use spreadsheet software your calculator to generate about 75 values of the table you started in

Question 3. The spreadsheet allows you to use a recursive rule to generate the data.

**5.** The agar dish that the MRSA bacteria are growing in has an area of 1,000 square

millimeters. The growth of the bacteria is limited in the lab by the size of the agar dish.

The growth of the bacteria can still be modeled by a proportional difference equation,

but now the rate of increase of the bacteria’s area is directly proportional to the

bacteria’s area and the difference between the agar dish’s area and the fungus’s area.

This constant of proportionality is the ratio of the original constant of proportionality

(0.2) and the maximum area the bacteria can reach (1,000 square millimeters). The

difference equation can be written as follows.

$$\frac{∆A}{∆t}=\frac{.2}{1000}A\left(1000-A\right)$$

Let Δ***t*** = 1 to simplify your work, and then use the difference equation to find the new

values of ***A***. Use a spreadsheet to calculate about ~~75~~ 8 values.

**6.** Compare the data generated in the unrestricted and restricted growth models. Record

your observations.

**7.** Use spreadsheet software or your graphing calculator to make a graph of the restricted

growth model. Sketch the graph below. What observations can you make about the

graph?

**8. REFLECTION**: The unrestricted bacteria growth models exponential growth and has a

common ratio of 1.2. Use the spreadsheet to find the ratio between successive values

in the restricted growth model. What do you notice? How does this support the graph

in Question 7?

**9.** A rancher has decided to dedicate a 400-square-mile portion of his ranch as a black bear

habitat. Working with his state, he plans to bring 10 young black bears to the habitat in

an effort to grow the population. His research shows that the annual growth rate of black

bears is about 0.8. Black bears thrive when the population density is no more than about

1.5 black bears per square mile.

**a.** What is the maximum sustainable number of black bears for the habitat?

**b.** Write a recursive rule showing the restricted growth in population for the black bears.

(***Hint:*** The constant of proportionality is the ratio of the unrestricted growth rate and

the maximum sustainable population.)

**c.** Make a table and graph showing the yearly population of the black bears in the

habitat. (Include enough years to show the population reaching the maximum

sustainable population.)

**d.** When will the population of bears in the habitat reach 500?

**e.** The rancher wants to repopulate the state with black bears. The rancher’s original

plan was to release the bears from his ranch when the population reaches 500. Do you

think this is a good decision based on the growth rate within the habitat over time? If

you agree with the rancher, support the decision with your data and graph. If you

disagree, propose a different target population value to the rancher; again, support

your proposal with the data and graph.