Name: $\qquad$ Date: $\qquad$

## Weighted Averages in Sports

## Diagnostic Assessment

For the past 12 school days, Mai has recorded how long her bus rides to school take in minutes. The times she recorded are shown in the table.

| 9 | 8 | 6 | 9 | 10 | 7 | 6 | 12 | 9 | 8 | 10 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

1. Find the mean for Mai's data. Show your reasoning.
2. In this situation, what does the mean tell us about Mai's trip to school?
3. For 5 days, Tyler has recorded how long his walks to school take in minutes. The mean for his data is 11 minutes. Without calculating, predict if each of the data sets shown could be Tyler's. Explain your reasoning.

- data set A: 11, 8, 7, 9, 8
- data set B: 12, 7, 13, 9, 14
- data set C: 11, 20, 6, 9, 10
- data set D: 8, 10, 9, 11, 11

4. Determine which data set is Tyler's. Explain how you know.

## Engage

## Calculating a Weighted Average

| Grading System I | Grading System II |
| :---: | :---: |
| Test Average - 40\% | Test Average - 60\% |
| Final Exam Grade - 25\% | Final Exam Grade - 15\% |
| Homework - 25\% | Homework - 15\% |
| Class Participation - 10\% | Class Participation - 10\% |

Look at the above grading system.

1. What do you notice? What do you wonder?
2. Which grading system would you prefer and why?
3. If your values are the following, which grading system do you prefer and why?

- Test average $=84$
- Final exam grade $=68$
- Homework = 90
- Class participation $=95$

4. If you score 10 points higher on the final exam, how does your final grade average change under each system?
5. If you score 6 points lower on the final exam, how does your final grade average change under each system? Which system is better for you?
6. Use the following information to find your final course average in each grading system.

- Test grades $\{80,74,82,88\}$
- Final exam grade $=84$
- Homework = 90
- Class participation $=95$

7. What weights would you assign to each component to set up a grading system? Each weight must be at least $10 \%$. Why do you think your grading system would be fair and effective?

## Explore

## Part 1: Batter's Slugging Average

1. How do you compare different baseball players?
2. What statistics might you look at?

One example of a weighted average in sports is a batter's slugging average (or percentage) in baseball. The slugging average (SLG) is calculated using the following equation.

$$
S L G=\frac{(1 \cdot S)+(2 \cdot D)+(3 \cdot T)+(4 \cdot H R)}{A B}
$$

where $S=$ singles, $D=$ doubles, $T=$ triples, $H R=$ home runs, and $A B=$ total at-bats. Each single has a weight of 1 , each double has a weight of 2 , each triple a weight of 3 , and each home run a weight of 4 . An at-bat without a hit has a weight of 0 .

In his first season with the New York Yankees, Babe Ruth set a record for slugging average that stood for more than 80 years. In 1920, Ruth pounded 172 in 458 at-bats. His hits consisted of 73 singles, 36 doubles, 9 triples, and 54 home runs, resulting in a total base count of $(73 \bullet 1)+$ $(36 \cdot 2)+(9 \bullet 3)+(54 \bullet 4)=388$. When his total number of bases, 388, is divided by his total at-bats, 458 , the result is 0.847 , his slugging percentage for the season. This record was broken in 2001 by Barry Bonds, who had 411 total base hits in 476 at-bats for a slugging average of 0.863 (statistics from www.baseball-almanac.com)
3. Find the slugging average for a player with the following statistics:

- $S=68$
- $\mathrm{D}=40$
- $\mathrm{T}=4$
- $\mathrm{HR}=16$
- $\mathrm{AB}=320$

4. Find the statistics of your favorite Major League baseball player in order to calculate his slugging average. Justify your answer.

## Part 2: Quarterback Rating

The National Football League (NFL) rates quarterbacks for statistical purposes against a fixed performance standards based on the statistical achievement of all qualified pro passers since 1960. This system allows passing performance to be compared from one season to the next. The following categories are used to compute the quarterback rating:

- percent of completions per attempt (\%COMP)
- percent of touchdown passes per attempt (\%TD)
- percent of interceptions per attempt (\%INT)
- average yards gained per attempt (YD)

The following is the simplified formula to compiling the quarterback rating (QR):

$$
Q R=\frac{25+10(\% C O M P)+40(\% T D)-50(\% I N T)+50(Y D)}{12}
$$

Note: This formula is subject to a few conditions discussed after Question 2.

1. For the first two games of the 2008 season, Dallas Cowboys quarterback Tony Romo completed 45 passes in 62 attempts for a total of 632 yards, with 4 touchdowns and 2 interceptions. Verify that Romo's quarterback rating for those games is approximately 113. Round each value to the nearest tenth.

- percent of completions per attempt (\%COMP) =
- percent of touchdown passes per attempt $(\% T D)=$
- percent of interceptions per attempt (\%INT) =
- average yards gained per attempt (YD) =

2. As of 2009, Steve Young has the highest career quarterback rating in NFL history. He completed 2,059 passes in 3,192 attempts for a total of 25,479 yards, with 174 touchdowns and 85 interceptions. Find Young's career quarterback rating.

The actual formula for the quarterback rating is subject to some conditions, represented below.

$$
Q R=\left[\frac{5(\% C O M P-30)}{6}\right]+\left[\frac{10(\% T D)}{3}\right]+\left[\frac{25(19-2(\% I N T))}{12}\right]+\left[\frac{25(Y D-3)}{6}\right]
$$

where it is understood that each item in the brackets is truncated to be no smaller than 0 and no larger than $475 / 12$ (given a maximum rating of $475 / 3$ or 158.3 ). This rating is rounded to the nearest tenth.

These truncated values mean there is a minimum and maximum value for each component of the rating.

For example, if
$\frac{10(\% T D)}{3} \leq \frac{475}{12}$, then $\% T D \leq \frac{475 \cdot 3}{12 \cdot 10}=11.875 \%$.
3. Show that $30 \% \leq \% \mathrm{COMP} \leq 77.5 \%$.
4. If $c \leq Y D \leq d$, find the values of $c$ and $d$.
5. If $0 \leq \% \mathrm{INT} \leq e$, find the value of $e$.
6. Explain why the actual formula and the simplified formula are the same when each component is between its minimum and maximum possible value.
7. Choose your favorite quarterback. Compute his quarterback rating based on this past year's statistics.

## Apply

1. Compare your favorite baseball player to five of your classmates' players. Who is the best player? Why?
2. Compare those players to the successful teams in the season. Did their slugging average lead to winning seasons for their teams? What other factors contribute to winning a baseball game?
3. Compare your favorite football quarterback to five of your classmates' players. Who is the best player? Why?
4. Compare those players to the successful teams in the season. Did the quarterback play lead to winning seasons for their teams? What other factors contribute to winning a football game?

## Test and Reflect

An index is a numerical scale. Characteristics of an index can be used for the following:

- to compare variables with one another or a reference number
- to give information about general trends
- to help make comparisons and judgments.

It is often calculated as a weighted sum of various factors resulting in a single summary number. How can the quarterback rating system be thought of as an index?

