Advanced Mathematical Decision Making Unit 1
Georgia's K-12 Mathematics Standards

Name: $\qquad$ Date: $\qquad$

# Pick a Number 

## Diagnostic Assessment

## UPC Codes

The first patent for barcodes was issued to Bernard Silver and Norman Woodland in 1952. Since then these coding systems have expanded, been modified, and applied to a variety of areas. One of the most common uses is in retail and grocery stores. Althougth the barcode is not the price of the item, it does allow for the item to be registered with an associated price. When the barcode is scanned, the associated price will be read by the cash register. Other uses are found in monitoring blood supplies, identification on prescription drugs, book checkout at libraries, tracking luggage, a nd express shipping services.

Another advantage to the barcode system is that when an additional digit is included, known as the check digit, many errors that occur during data entry can be detected. These errors occur when passing information over the phone or internet. It is quite easy for people to transpose numbers (45 when it should be 54), replace a single digit with another, omit digits, or double an incorrect digit (799 is entered as 779). Using a check digit within a check equation helps to catch these errors and verify the validity of the number. As you will see in the ISBN barcode, the number is also used for identification purposes.

Begin by taking samples of barcodes that you have collected from items that are sold in stores. This type of barcode is referred to as the Universal Product Code, or UPC.
For the first example, use the given UPC symbol 7-86936-24425-0 from the movie "The Incredibles."

To verify this number, follow the steps:

- Every even-positioned digit, counting from the right to left, will be multiplied by 3. All oddpositioned digits will be multiplied by 1 .
- Sum the products
- Determine the validity by dividing the sum by 10 . If the remainder is 0 , the UPC number is valid.

1. Is the UPC number valid for "The Incredibles" movie?
2. This time, try to figure out the check digit in the following UPC number:

7-96714-78601-y, where $y$ is the check digit.

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## Engage

Identification numbers are present everywhere in society. Today's identification numbers are more sophisticated than those introduced years earlier (For example, Social Security numbers). Today's numbers have a check digit to partially ensure that they have been correctly scanned or entered into a computer. Universal Product Codes (UPCs), typically in the form of barcodes, identify retail products.


The 12-digit UPC barcode consists of three parts:

- manufacturer number,
- product number, and
- check digit.

For example, the manufacturer number for the Dr. Pepper Company is 078000 and appears in the first six digits of all of the company's product UPC barcodes. GS1, formerly the Uniform Code Council, issues a company this six-digit number.

Every item sold by a company requires a different five-digit product number. This includes specific products, their different sizes, their array of colors, their variety of flavors, and other distinguishing features.

The last number is the check digit, which guards against entry errors and fraud.
The check digit in a UPC number (that is, the twelfth digit) is determined in the following manner:

- Multiply the first digit by 3.
- Add the second digit
- Multiply the third digit by 3 .
- Add the fourth digit.
- Continue this alternating process for the Digits 5 to 12.

1. Using the internet, can you find other manufactured products using their UPC number?

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Read this article How DO UPC Barcodes Work and How to Get Them, then answer the following questions.
2. Who issues a bar code?
3. Is the price of an item included in the barcode?
4. Can a barcode be a different color?
5. What is the cost of a barcode?

## Explore

The check digit is chosen so that the calculation described previously totals a number whose final digit is 0 . In the UPC number $a_{1} a_{2} a_{3} a_{4} a_{5} a_{6} a_{7} a_{8} a_{9} a_{10} a_{11} d$, the check digit is $d$, for which the sum
$3 a_{1}+a_{2}+3 a_{3}+a_{4}+3 a_{5}+a_{6}++3 a_{7}+a_{8}+3 a_{9}+a_{10}+3 a_{11}+d$ ends in 0.
In this weighted sum, the weights are: $\{3,1,3,1,3,1,3,1,3,1,3,1\}$. When entering a code number, the single-digit error is most common (For example, keying in 8 instead of 3 ). Another common error is the transposition error, where the order of two adjacent digits is reversed (for example, writing 83 instead of 38). Systems have been established to detect and correct (when possible) these and other errors almost immediately.

1. Show that 0-58200-48826-5 is a valid UPC number.
2. Show that 0-52200-48826-5 is an invalid UPC number. If someone made a single-digit error when entering this invalid number, can you tell which digit is incorrect? Why or why not? Change one digit in this invalid number so the resulting number is valid.
3. Determine the check digit (d) for the UPC number 38137009213 d .
4. Suppose you entered 8 instead of 9 when recording the UPC number 155210021496. Explain why the UPC method will detect this error. Do you think the UPC method will detect all such single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.
5. Suppose you use the weights $\{7,1,7,1,7,1,7,1,7,1,7,1\}$ instead of $\{3,1,3,1,3,1$, $3,1,3,1,3,1\}$ in the UPC method. Do you think this altered method will detect single digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.

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## Apply

Identification numbers are present everywhere in society. Today's identification numbers are more sophisticated than those introduced years earlier (for example, Social Security numbers).

Today's numbers have a check digit to partially ensure that they have been correctly scanned or entered into a computer.

Credit cards have 16-digit numbers, of which the first 15 digits identify the credit card and the sixteenth digit is the check digit. The following figure shows the significance of the digits:


```
1234 5678 9012 1314
\begin{tabular}{ccc} 
MII & Issuer & Account
\end{tabular} Check
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MII stands for major industry identifier; 4 and 5 indicate "Banking and Financial." VISA cards begin with 4 and MasterCard cards with 5.

1. MasterCard numbers begin with $51,52,53,54$, or 55 . What is the maximum number of credit cards that MasterCard can issue?

A check digit is used to help validate credit card numbers. The credit card companies use the Codabar method to determine the check digit.

This method consists of the following steps:

- Add the digits in the odd-numbered positions and double this total.
- Add the number of odd-position digits that are more than 4 to the total.
- Add the even-position digits.
- Choose a check digit that makes this calculation total a number whose final digit is 0 .

Libraries, shipping/receiving companies, and blood banks also use the Codabar method.
2. Show that the check digit (d) for the VISA card 416200123456 789d is 3.
3. What is the check digit (d) for the MasterCard number 542498132720 008d?
4. Show that 4128001243890110 is an invalid VISA credit card number. If someone made a single-digit error when entering this invalid number, can you tell which digit is incorrect? Why or why not?

The following is another way to explain the Codabar method:

- If a digit is in an even-numbered position, add it to the total.
- If a digit is in an odd-numbered position, multiply it by 2.
- If the product is equal to or greater than 10 , subtract 9 from the product.
- Add this difference to the total.
- After the first 15 digits have been processed, choose the check digit so that the sum of the 16 digits ends in 0 .

5. Explain why both methods yield the same check digit.
6. Suppose you entered 8 instead of 9 when recording the credit card number 40012560 0196 4310. Explain why the Codabar method will detect this error. Do you think the Codabar method will detect all such single-digit errors using a method similar to the one for showing all such errors are detected for UPC numbers? Either write Yes or give several examples of numbers with an error that will not be detected.
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## Test and Reflect

The Codabar method detects $98 \%$ of all transposition errors. There is one pair of numbers, however, that when transposed will not be detected by the Codabar method.

1. Find this pair of numbers and explain why this transposition error will go undetected. (Hint: There are several cases to consider depending on the positions of the transposed digits.)
2. In what ways is the Codabar method like "Indices Using Weighted Sums and Averages"?

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